# Summary: An Ideal Model for Polymorphic Recursive Types (MacQueen et al.)

Jun Inoue

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Type systems are often prescribed by an operational semantics, i.e. the typing/type inference rules. This paper gives a denotational semantics for a Hindley-Milner style type system.

The denotation is based on the common intuition that a type (or its denotation) is the set of values that can be given that type. However, allowing arbitrary sets to be types is undesirable because some sets can behave badly. One of the problems is that we cannot allow recursive types (types defined by a recursive equation, such as "the type $\sigma$ s.t. $\sigma = \sigma \to t$") since the defining equation may not have a solution.

This paper shows a denotational type system which requires a set to be an ideal in order to be a type. By imposing this condition, the type universe forms a complete ultrametric space, which guarantees that, (with mild modifications to the type expression grammar) all recursive type equations have a unique solution.